UNIT 1

Real Numbers, Exponents, and Scientific Notation

MODULE 1

Real Numbers

FL 8.NS.1.1, 8.NS.1.2, 8.EE.1.2

MODULE 2

Exponents and Scientific Notation

FL 8.EE.1.1, 8.EE.1.3, 8.EE.1.4

CAREERS IN MATH

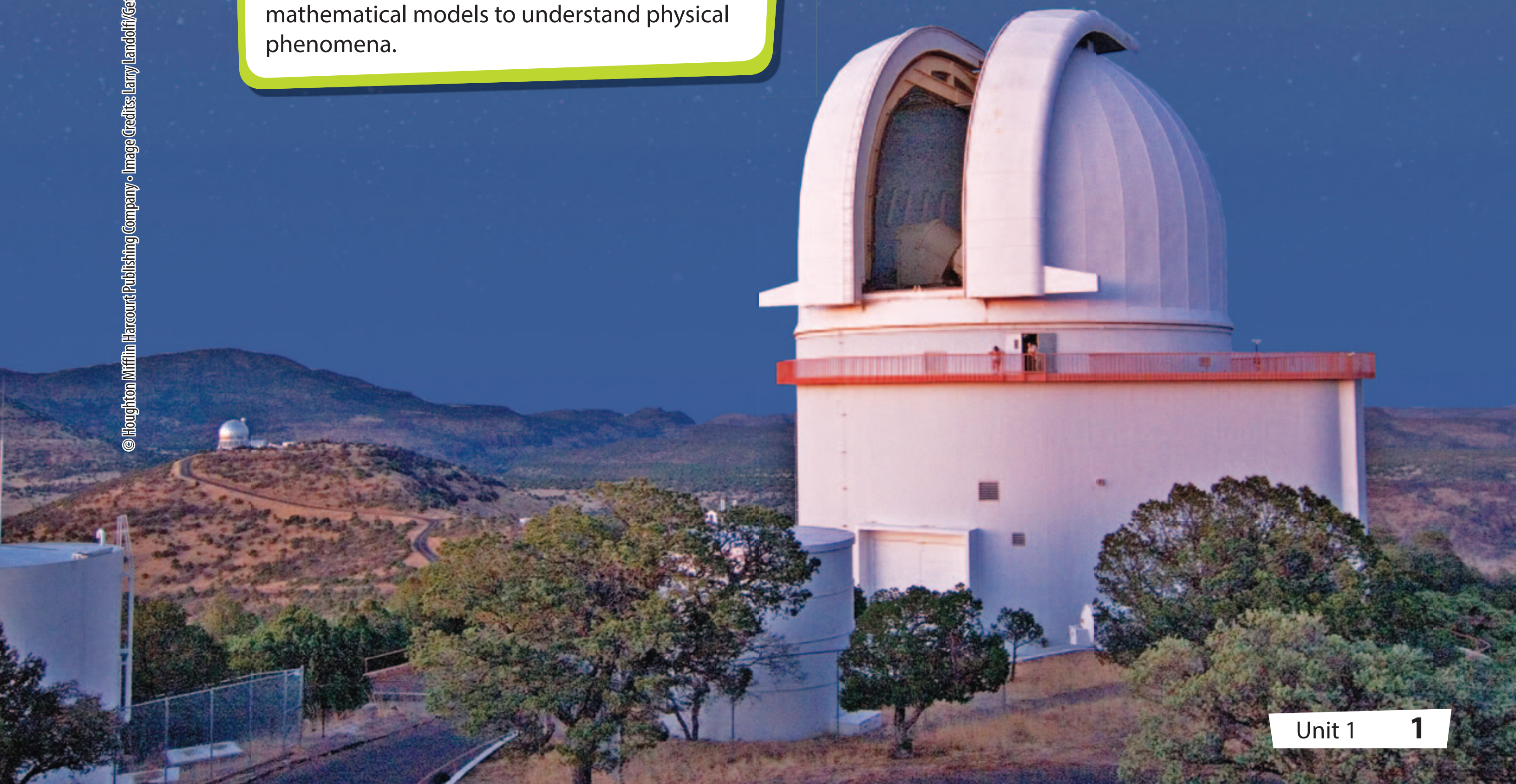
Astronomer An astronomer is a scientist who studies and tries to interpret the universe beyond Earth. Astronomers use math to calculate distances to celestial objects and to create mathematical models to help them understand the dynamics of systems from stars and planets to black holes. If you are interested in a career as an astronomer, you should study the following mathematical subjects:

- Algebra
- Geometry
- Trigonometry
- Calculus

Research other careers that require creating mathematical models to understand physical phenomena.

Unit 1 Performance Task

At the end of the unit, check out how **astronomers** use math.



C $729 = x^3$

$\sqrt[3]{729} = \sqrt[3]{x^3}$ — *Solve for x by taking the cube root of both sides.*

$\sqrt[3]{729} = x$ — *Apply the definition of cube root.*

$9 = x$ — *Think: What number cubed equals 729?*

The solution is 9.

D $x^3 = \frac{8}{125}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{8}{125}}$ — *Solve for x by taking the cube root of both sides.*

$x = \sqrt[3]{\frac{8}{125}}$ — *Apply the definition of cube root.*

$x = \frac{2}{5}$ — *Think: What number cubed equals $\frac{8}{125}$?*

The solution is $\frac{2}{5}$.

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YOUR TURN

Solve each equation for *x*.

7. $x^2 = 196$ ________

8. $x^2 = \frac{9}{256}$ ________

9. $x^3 = 512$ ________

10. $x^3 = \frac{64}{343}$ ________

EXPLORE ACTIVITY FL 8.NS.1.2, 8.EE.1.2

Estimating Irrational Numbers

Irrational numbers are numbers that are not rational. In other words, they cannot be written in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not 0. Square roots of perfect squares are rational numbers. Square roots of numbers that are not perfect squares are irrational. The number $\sqrt{3}$ is irrational because 3 is not a perfect square of any rational number.

Estimate the value of $\sqrt{2}$.

A Since 2 is not a perfect square, $\sqrt{2}$ is irrational.

B To estimate $\sqrt{2}$, first find two consecutive perfect squares that 2 is between. Complete the inequality by writing these perfect squares in the boxes.

$\boxed{} < 2 < \boxed{}$

C Now take the square root of each number.

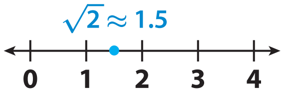
$\sqrt{\boxed{}} < \sqrt{2} < \sqrt{\boxed{}}$

D Simplify the square roots of perfect squares.

$\boxed{} < \sqrt{2} < \boxed{}$

$\sqrt{2}$ is between ______ and ______.

E Estimate that $\sqrt{2} \approx 1.5$.



F To find a better estimate, first choose some numbers between 1 and 2 and square them. For example, choose 1.3, 1.4, and 1.5.

$1.3^2 =$ ______ $1.4^2 =$ ______ $1.5^2 =$ ______

Is $\sqrt{2}$ between 1.3 and 1.4? How do you know?

____

____

Is $\sqrt{2}$ between 1.4 and 1.5? How do you know?

____

____

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

G Locate and label this value on the number line.

Reflect

11. How could you find an even better estimate of $\sqrt{2}$?

____

____

12. Find a better estimate of $\sqrt{2}$. Draw a number line and locate and label your estimate.

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

13. Estimate the value of $\sqrt{7}$ to two decimal places. Draw a number line and locate and label your estimate.

$\sqrt{7}$ is between ______ and ______, so $\sqrt{7} \approx$ ______.

Guided Practice

Write each fraction or mixed number as a decimal. (Example 1)

1. $\frac{2}{5}$ ______

2. $\frac{8}{9}$ ______

3. $3\frac{3}{4}$ ______

4. $\frac{7}{10}$ ______

5. $2\frac{3}{8}$ ______

6. $\frac{5}{6}$ ______

Write each decimal as a fraction or mixed number in simplest form. (Example 2)

7. 0.675 ______

8. 5.6 ______

9. 0.44 ______

10. $0.\overline{4}$

$$\begin{aligned} 10x &= \square \\ -x \quad &- \square \\ \hline \square\, x &= \square \end{aligned}$$

$x =$ ______

11. $0.\overline{26}$

$$\begin{aligned} 100x &= \square \\ -x \quad &- \square \\ \hline \square\, x &= \square \end{aligned}$$

$x =$ ______

12. $0.\overline{325}$

$$\begin{aligned} 1000x &= \square \\ -x \quad &- \square \\ \hline \square\, x &= \square \end{aligned}$$

$x =$ ______

Solve each equation for *x*. (Example 3)

13. $x^2 = 144$

$x = \pm\sqrt{\square} = \pm\square$

14. $x^2 = \frac{25}{289}$

$x = \pm\sqrt{\frac{\square}{\square}} = \pm\frac{\square}{\square}$

15. $x^3 = 216$

$x = \sqrt[3]{\square} = \square$

Approximate each irrational number to two decimal places without a calculator.
(Explore Activity)

16. $\sqrt{5} \approx \square$

17. $\sqrt{3} \approx \square$

18. $\sqrt{10} \approx \square$

ESSENTIAL QUESTION CHECK-IN

19. What is the difference between rational and irrational numbers?

______

______

Name ____________________ Class ____________ Date ____________

1.1 Independent Practice

FL 8.NS.1.1, 8.NS.1.2, 8.EE.1.2

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20. A $\frac{7}{16}$-inch-long bolt is used in a machine. What is the length of the bolt written as a decimal?

____

21. The weight of an object on the moon is $\frac{1}{6}$ its weight on Earth. Write $\frac{1}{6}$ as a decimal.

____

22. The distance to the nearest gas station is $2\frac{4}{5}$ kilometers. What is this distance written as a decimal?

____

23. A baseball pitcher has pitched $98\frac{2}{3}$ innings. What is the number of innings written as a decimal?

____

24. A heartbeat takes 0.8 second. How many seconds is this written as a fraction?

____

25. There are 26.2 miles in a marathon. Write the number of miles using a fraction.

____

26. The average score on a biology test was $72.\overline{1}$. Write the average score using a fraction.

____

27. The metal in a penny is worth about 0.505 cent. How many cents is this written as a fraction?

____

28. **Multistep** An artist wants to frame a square painting with an area of 400 square inches. She wants to know the length of the wood trim that is needed to go around the painting.

a. If *x* is the length of one side of the painting, what equation can you set up to find the length of a side? ______________________

b. Solve the equation you wrote in part a. How many solutions does the equation have?

____

____

c. Do all of the solutions that you found in part b make sense in the context of the problem? Explain.

____

____

____

d. What is the length of the wood trim needed to go around the painting?

____

29. **Analyze Relationships** To find $\sqrt{15}$, Beau found $3^2 = 9$ and $4^2 = 16$. He said that since 15 is between 9 and 16, $\sqrt{15}$ must be between 3 and 4. He thinks a good estimate for $\sqrt{15}$ is $\frac{3+4}{2} = 3.5$. Is Beau's estimate high, low, or correct? Explain.

30. **Justify Reasoning** What is a good estimate for the solution to the equation $x^3 = 95$? How did you come up with your estimate?

31. The volume of a sphere is 36π ft^3. What is the radius of the sphere? Use the formula $V = \frac{4}{3}\pi r^3$ to find your answer.

FOCUS ON HIGHER ORDER THINKING

Work Area

32. **Draw Conclusions** Can you find the cube root of a negative number? If so, is it positive or negative? Explain your reasoning.

33. **Make a Conjecture** Evaluate and compare the following expressions.

$$\sqrt{\frac{4}{25}} \text{ and } \frac{\sqrt{4}}{\sqrt{25}} \qquad \sqrt{\frac{16}{81}} \text{ and } \frac{\sqrt{16}}{\sqrt{81}} \qquad \sqrt{\frac{36}{49}} \text{ and } \frac{\sqrt{36}}{\sqrt{49}}$$

Use your results to make a conjecture about a division rule for square roots. Since division is multiplication by the reciprocal, make a conjecture about a multiplication rule for square roots.

34. **Persevere in Problem Solving** The difference between the solutions to the equation $x^2 = a$ is 30. What is a? Show that your answer is correct.

LESSON

1.2 Sets of Real Numbers

FL 8.NS.1.1

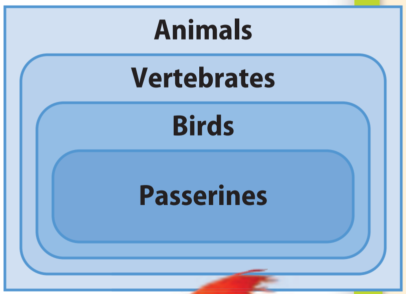
Know that numbers that are not rational are called irrational. . . .

ESSENTIAL QUESTION

How can you describe relationships between sets of real numbers?

Classifying Real Numbers

Biologists classify animals based on shared characteristics. A cardinal is an animal, a vertebrate, a bird, and a passerine.



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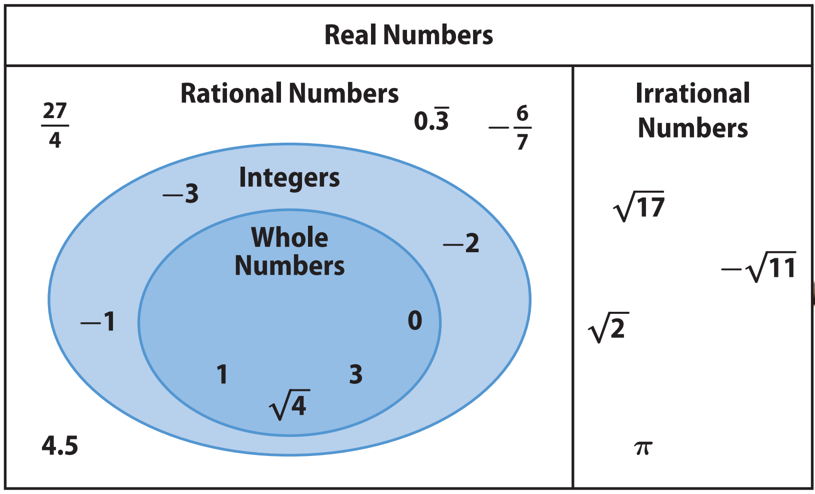
You already know that the set of rational numbers consists of whole numbers, integers, and fractions. The set of **real numbers** consists of the set of rational numbers and the set of irrational numbers.



Passerines, such as the cardinal, are also called "perching birds."

EXAMPLE 1

FL 8.NS.1.1

Write all names that apply to each number.

A $\sqrt{5}$ — 5 is a whole number that is not a perfect square.

irrational, real

B -17.84 — -17.84 is a terminating decimal.

rational, real

C $\frac{\sqrt{81}}{9}$ — $\frac{\sqrt{81}}{9} = \frac{9}{9} = 1$

whole, integer, rational, real

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Math Talk
Mathematical Practices

What types of numbers are between 3.1 and 3.9 on a number line?

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YOUR TURN

Write all names that apply to each number.

1. A baseball pitcher has pitched $12\frac{2}{3}$ innings.

____

2. The length of the side of a square that has an area of 10 square yards. ______________

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Understanding Sets and Subsets of Real Numbers

By understanding which sets are subsets of types of numbers, you can verify whether statements about the relationships between sets are true or false.

EXAMPLE 2

FL 8.NS.1.1

Tell whether the given statement is true or false. Explain your choice.

A All irrational numbers are real numbers.

True. Every irrational number is included in the set of real numbers. Irrational numbers are a subset of real numbers.

B No rational numbers are whole numbers.

False. A whole number can be written as a fraction with a denominator of 1, so every whole number is included in the set of rational numbers. Whole numbers are a subset of rational numbers.

Math Talk
Mathematical Practices
Give an example of a rational number that is a whole number. Show that the number is both whole and rational.

YOUR TURN

Tell whether the given statement is true or false. Explain your choice.

3. All rational numbers are integers.

____

____

____

4. Some irrational numbers are integers.

____

____

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Identifying Sets for Real-World Situations

Real numbers can be used to represent real-world quantities. Highways have posted speed limit signs that are represented by natural numbers such as 55 mph. Integers appear on thermometers. Rational numbers are used in many daily activities, including cooking. For example, ingredients in a recipe are often given in fractional amounts such as $\frac{2}{3}$ cup flour.

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EXAMPLE 3 Real World

FL 8.NS.1.1

Identify the set of numbers that best describes each situation. Explain your choice.

A the number of people wearing glasses in a room

The set of whole numbers best describes the situation. The number of people wearing glasses may be 0 or a counting number.

B the circumference of a flying disk has a diameter of 8, 9, 10, 11, or 14 inches

The set of irrational numbers best describes the situation. Each circumference would be a product of π and the diameter, and any multiple of π is irrational.

My Notes

YOUR TURN

Identify the set of numbers that best describes the situation. Explain your choice.

5. the amount of water in a glass as it evaporates

____

____

____

6. the number of seconds remaining when a song is playing, displayed as a negative number

____

____

____

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Guided Practice

Write all names that apply to each number. (Example 1)

1. $\frac{7}{8}$

2. $\sqrt{36}$

3. $\sqrt{24}$

4. 0.75

5. 0

6. $-\sqrt{100}$

7. $5.\overline{45}$

8. $-\frac{18}{6}$

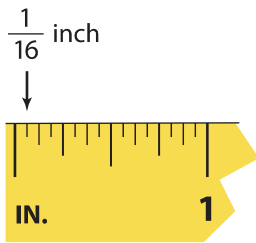
Tell whether the given statement is true or false. Explain your choice. (Example 2)

9. All whole numbers are rational numbers.

10. No irrational numbers are whole numbers.

Identify the set of numbers that best describes each situation. Explain your choice. (Example 3)

11. the change in the value of an account when given to the nearest dollar



12. the markings on a standard ruler

ESSENTIAL QUESTION CHECK-IN

13. What are some ways to describe the relationships between sets of numbers?

Name ____________________ Class ____________ Date ____________

1.2 Independent Practice

FL 8.NS.1.1

Write all names that apply to each number. Then place the numbers in the correct location on the Venn diagram.

14. $\sqrt{9}$ ____________________

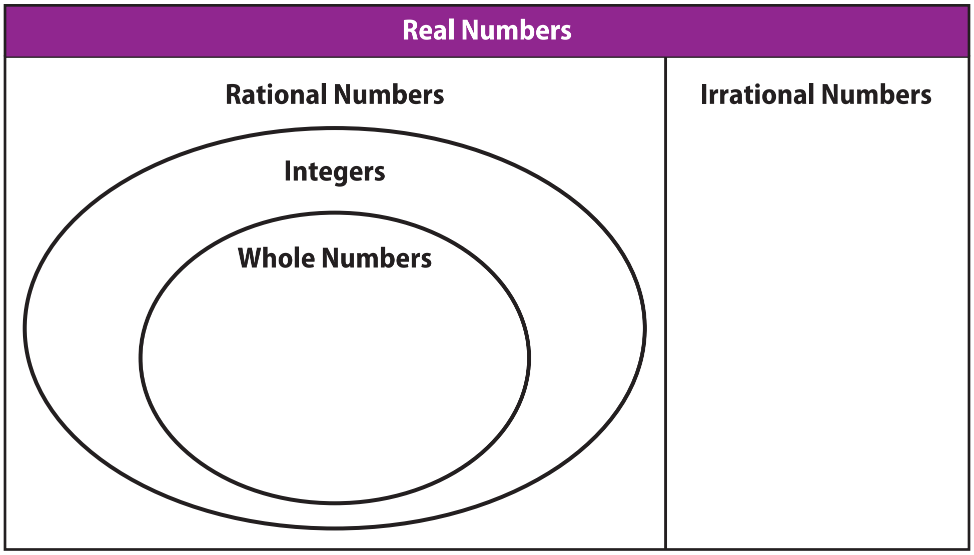
15. 257 ____________________

16. $\sqrt{50}$ ____________________

17. $8\frac{1}{2}$ ____________________

18. 16.6 ____________________

19. $\sqrt{16}$ ____________________



Identify the set of numbers that best describes each situation. Explain your choice.

20. the height of an airplane as it descends to an airport runway

____

21. the score with respect to par of several golfers: 2, −3, 5, 0, −1

____

____

22. **Critique Reasoning** Ronald states that the number $\frac{1}{11}$ is not rational because, when converted into a decimal, it does not terminate. Nathaniel says it is rational because it is a fraction. Which boy is correct? Explain.

____

____

UNIT 1

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.

1. TCREEFP SEAQUR ☐☐☐☐☐☐◯ ☐◯☐☐☐☐

2. NOLRATAI RUNMEB ◯☐☐☐◯☐☐☐ ☐☐☐☐☐☐

3. PERTIANEG MALCEDI ☐☐☐☐☐☐☐☐☐ ☐☐☐☐☐◯☐

4. LAER SEBMNUR ◯☐☐☐ ☐◯☐☐☐◯☐

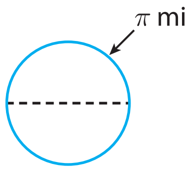
5. NIISICFTCE OITANTON ◯☐☐◯☐☐☐☐☐☐ ☐☐☐☐☐☐◯☐

1. Has integers as its square roots. (Lesson 1.1)
2. Any number that can be written as a ratio of two integers. (Lesson 1.1)
3. A decimal in which one or more digits repeat infinitely. (Lesson 1.1)
4. The set of rational and irrational numbers. (Lesson 1.2)
5. A method of writing very large or very small numbers by using powers of 10. (Lesson 2.2)

Q: What keeps a square from moving?

A: __ __ __ __ __ __   __ __ __ __ __!

23. **Critique Reasoning** The circumference of a circular region is shown. What type of number best describes the diameter of the circle? Explain your answer. ______



24. **Critical Thinking** A number is not an integer. What type of number can it be?

______

25. A grocery store has a shelf with half-gallon containers of milk. What type of number best represents the total number of gallons?

______

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

26. **Explain the Error** Katie said, "Negative numbers are integers." What was her error?

______

______

27. **Justify Reasoning** Can you ever use a calculator to determine if a number is rational or irrational? Explain.

______

______

______

______

______

______

______

28. **Draw Conclusions** The decimal $0.\overline{3}$ represents $\frac{1}{3}$. What type of number best describes $0.\overline{9}$, which is $3 \cdot 0.\overline{3}$? Explain.

______

29. **Communicate Mathematical Ideas** Irrational numbers can never be precisely represented in decimal form. Why is this?

______

______

______

______

LESSON 1.3

Ordering Real Numbers

FL 8.NS.1.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

ESSENTIAL QUESTION

How do you order a set of real numbers?

Comparing Irrational Numbers

Between any two real numbers is another real number. To compare and order real numbers, you can approximate irrational numbers as decimals.

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EXAMPLE 1

FL 8.NS.1.2

Compare $\sqrt{3} + 5$ ◯ $3 + \sqrt{5}$. Write <, >, or =.

Use perfect squares to estimate square roots.
$1^2 = 1 \quad 2^2 = 4 \quad 3^2 = 9$

STEP 1 First approximate $\sqrt{3}$.

$\sqrt{3}$ is between 1 and 2.

Next approximate $\sqrt{5}$.

$\sqrt{5}$ is between 2 and 3.

STEP 2 Then use your approximations to simplify the expressions.

$\sqrt{3} + 5$ is between 6 and 7.

$3 + \sqrt{5}$ is between 5 and 6.

So, $\sqrt{3} + 5 > 3 + \sqrt{5}$.

My Notes

Reflect

1. If $7 + \sqrt{5}$ is equal to $\sqrt{5}$ plus a number, what do you know about the number? Why?

____

2. What are the closest two integers that $\sqrt{300}$ is between?

____

YOUR TURN

Compare. Write <, >, or =.

3. $\sqrt{2} + 4$ ◯ $2 + \sqrt{4}$

4. $\sqrt{12} + 6$ ◯ $12 + \sqrt{6}$

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My Notes

Ordering Real Numbers

You can compare and order real numbers and list them from least to greatest.

EXAMPLE 2

FL 8.NS.1.2

Order $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ from least to greatest.

STEP 1 First approximate $\sqrt{22}$.

$\sqrt{22}$ is between 4 and 5. Since you don't know where it falls between 4 and 5, you need to find a better estimate for $\sqrt{22}$ so you can compare it to $4\frac{1}{2}$.

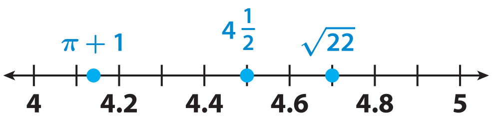
Since 22 is closer to 25 than 16, use squares of numbers between 4.5 and 5 to find a better estimate of $\sqrt{22}$.

$4.5^2 = 20.25$ $\quad$ $4.6^2 = 21.16$ $\quad$ $4.7^2 = 22.09$ $\quad$ $4.8^2 = 23.04$

Since $4.7^2 = 22.09$, an approximate value for $\sqrt{22}$ is 4.7.

An approximate value of π is 3.14. So an approximate value of $\pi + 1$ is 4.14.

STEP 2 Plot $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ on a number line.



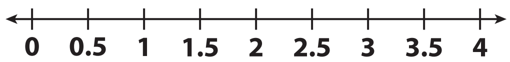
Read the numbers from left to right to place them in order from least to greatest.

From least to greatest, the numbers are $\pi + 1$, $4\frac{1}{2}$, and $\sqrt{22}$.

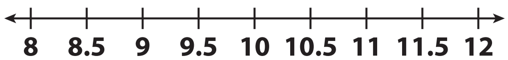
YOUR TURN

Order the numbers from least to greatest. Then graph them on the number line.

5. $\sqrt{5}$, 2.5, $\sqrt{3}$ ____________________



6. π^2, 10, $\sqrt{75}$ ____________________



Math Talk
Mathematical Practices

If real numbers a, b, and c are in order from least to greatest, what is the order of their opposites from least to greatest? Explain.

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Ordering Real Numbers in a Real-World Context

Calculations and estimations in the real world may differ. It can be important to know not only which are the most accurate but which give the greatest or least values, depending upon the context.

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EXAMPLE 3 Real World

FL 8.NS.1.2

Four people have found the distance in kilometers across a canyon using different methods. Their results are given in the table. Order the distances from greatest to least.

Distance Across Quarry Canyon (km)			
Juana	**Lee Ann**	**Ryne**	**Jackson**
$\sqrt{28}$	$\frac{23}{4}$	$5.\overline{5}$	$5\frac{1}{2}$

STEP 1 Write each value as a decimal.

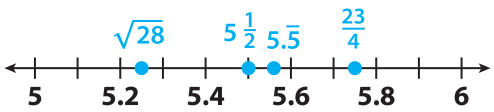
$\sqrt{28}$ is between 5.2 and 5.3. Since $5.3^2 = 28.09$, an approximate value for $\sqrt{28}$ is 5.3.

$\frac{23}{4} = 5.75$

$5.\overline{5}$ is 5.555…, so $5.\overline{5}$ to the nearest hundredth is 5.56.

$5\frac{1}{2} = 5.5$

STEP 2 Plot $\sqrt{28}$, $\frac{23}{4}$, $5.\overline{5}$, and $5\frac{1}{2}$ on a number line.



From greatest to least, the distances are:

$\frac{23}{4}$ km, $5.\overline{5}$ km, $5\frac{1}{2}$ km, $\sqrt{28}$ km.

YOUR TURN

7. Four people have found the distance in miles across a crater using different methods. Their results are given below.

Jonathan: $\frac{10}{3}$, Elaine: $3.\overline{45}$, José: $3\frac{1}{2}$, Lashonda: $\sqrt{10}$

Order the distances from greatest to least.

____

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Guided Practice

Compare. Write $<$, $>$, or $=$. (Example 1)

1. $\sqrt{3} + 2$ ◯ $\sqrt{3} + 3$

2. $\sqrt{11} + 15$ ◯ $\sqrt{8} + 15$

3. $\sqrt{6} + 5$ ◯ $6 + \sqrt{5}$

4. $\sqrt{9} + 3$ ◯ $9 + \sqrt{3}$

5. $\sqrt{17} - 3$ ◯ $-2 + \sqrt{5}$

6. $10 - \sqrt{8}$ ◯ $12 - \sqrt{2}$

7. $\sqrt{7} + 2$ ◯ $\sqrt{10} - 1$

8. $\sqrt{17} + 3$ ◯ $3 + \sqrt{11}$

9. Order $\sqrt{3}$, 2π, and 1.5 from least to greatest. Then graph them on the number line. (Example 2)

$\sqrt{3}$ is between ____________ and ____________, so $\sqrt{3} \approx$ ____________.

$\pi \approx 3.14$, so $2\pi \approx$ ____________.

0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7

From least to greatest, the numbers are ____________, ____________, ____________.

10. Four people have found the perimeter of a forest using different methods. Their results are given in the table. Order their calculations from greatest to least. (Example 3)

____

____

Forest Perimeter (km)			
Leon	Mika	Jason	Ashley
$\sqrt{17} - 2$	$1 + \frac{\pi}{2}$	$\frac{12}{5}$	2.5

ESSENTIAL QUESTION CHECK-IN

11. Explain how to order a set of real numbers.

____

____

____

____

____

Name ______ Class ______ Date ______

1.3 Independent Practice

FL 8.NS.1.2

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Order the numbers from least to greatest.

12. $\sqrt{7}, 2, \frac{\sqrt{8}}{2}$

______

13. $\sqrt{10}, \pi, 3.5$

______

14. $\sqrt{220}, -10, \sqrt{100}, 11.5$

______

15. $\sqrt{8}, -3.75, 3, \frac{9}{4}$

______

16. Your sister is considering two different shapes for her garden. One is a square with side lengths of 3.5 meters, and the other is a circle with a diameter of 4 meters.

a. Find the area of the square. ______

b. Find the area of the circle. ______

c. Compare your answers from parts **a** and **b**. Which garden would give your sister the most space to plant?

______

______

17. Winnie measured the length of her father's ranch four times and got four different distances. Her measurements are shown in the table.

Distance Across Father's Ranch (km)			
1	**2**	**3**	**4**
$\sqrt{60}$	$\frac{58}{8}$	$7.\overline{3}$	$7\frac{3}{5}$

a. To estimate the actual length, Winnie first approximated each distance to the nearest hundredth. Then she averaged the four numbers. Using a calculator, find Winnie's estimate.

______

______

b. Winnie's father estimated the distance across his ranch to be $\sqrt{56}$ km. How does this distance compare to Winnie's estimate?

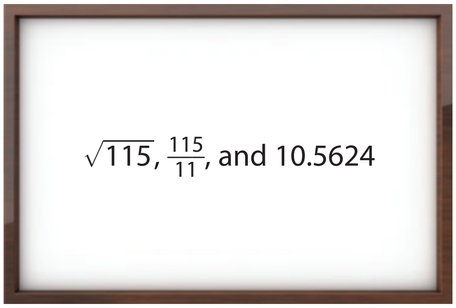
______

Give an example of each type of number.

18. a real number between $\sqrt{13}$ and $\sqrt{14}$ ______

19. an irrational number between 5 and 7 ______

20. A teacher asks his students to write the numbers shown in order from least to greatest. Paul thinks the numbers are already in order. Sandra thinks the order should be reversed. Who is right?



____

____

____

21. **Math History** There is a famous irrational number called Euler's number, symbolized with an *e*. Like π, its decimal form never ends or repeats. The first few digits of *e* are 2.7182818284.

a. Between which two square roots of integers could you find this number?

____

b. Between which two square roots of integers can you find π?

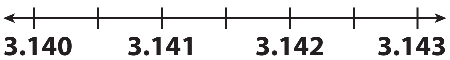
____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

22. **Analyze Relationships** There are several approximations used for π, including 3.14 and $\frac{22}{7}$. π is approximately 3.14159265358979…

a. Label π and the two approximations on the number line.



b. Which of the two approximations is a better estimate for π? Explain.

____

c. Find a whole number *x* so that the ratio $\frac{x}{113}$ is a better estimate for π than the two given approximations. ______________

23. **Communicate Mathematical Ideas** If a set of six numbers that include both rational and irrational numbers is graphed on a number line, what is the fewest number of distinct points that need to be graphed? Explain.

____

____

____

24. **Critique Reasoning** Jill says that $12.\overline{6}$ is less than 12.63. Explain her error.

____

MODULE QUIZ

Ready to Go On?

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1.1 Rational and Irrational Numbers

Write each fraction as a decimal or each decimal as a fraction.

1. $\frac{7}{20}$ ________ **2.** $1.\overline{27}$ ________ **3.** $1\frac{7}{8}$ ________

Solve each equation for *x*.

4. $x^2 = 81$ ________ **5.** $x^3 = 343$ ________ **6.** $x^2 = \frac{1}{100}$ ________

7. A square patio has an area of 200 square feet. How long is each side of the patio to the nearest 0.05? ________

1.2 Sets of Real Numbers

Write all names that apply to each number.

8. $\frac{121}{\sqrt{121}}$ ________

9. $\frac{\pi}{2}$ ________

10. Tell whether the statement "All integers are rational numbers" is true or false. Explain your choice.

____

____

1.3 Ordering Real Numbers

Compare. Write $<$, $>$, or $=$.

11. $\sqrt{8} + 3$ ◯ $8 + \sqrt{3}$ **12.** $\sqrt{5} + 11$ ◯ $5 + \sqrt{11}$

Order the numbers from least to greatest.

13. $\sqrt{99}$, π^2, $9.\overline{8}$ ________ **14.** $\sqrt{\frac{1}{25}}$, $\frac{1}{4}$, $0.\overline{2}$ ________

ESSENTIAL QUESTION

15. How are real numbers used to describe real-world situations?

____

____

MODULE 1 **MIXED REVIEW**

Assessment Readiness

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Selected Response

1. The square root of a number is 9. What is the other square root?

Ⓐ −9 Ⓒ 3
Ⓑ −3 Ⓓ 81

2. A square acre of land is 4,840 square yards. Between which two integers is the length of one side?

Ⓐ between 24 and 25 yards
Ⓑ between 69 and 70 yards
Ⓒ between 242 and 243 yards
Ⓓ between 695 and 696 yards

3. Which of the following is an integer but not a whole number?

Ⓐ −9.6 Ⓒ 0
Ⓑ −4 Ⓓ 3.7

4. Which statement is false?

Ⓐ No integers are irrational numbers.
Ⓑ All whole numbers are integers.
Ⓒ No real numbers are irrational numbers.
Ⓓ All integers greater than 0 are whole numbers.

5. Which set of numbers best describes the displayed weights on a digital scale that shows each weight to the nearest half pound?

Ⓐ whole numbers
Ⓑ rational numbers
Ⓒ real numbers
Ⓓ integers

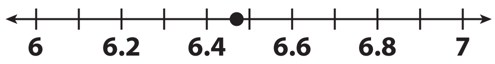
6. Which of the following is not true?

Ⓐ $\pi^2 < 2\pi + 4$ Ⓒ $\sqrt{27} + 3 > \frac{17}{2}$
Ⓑ $3\pi > 9$ Ⓓ $5 - \sqrt{24} < 1$

7. Which number is between $\sqrt{21}$ and $\frac{3\pi}{2}$?

Ⓐ $\frac{14}{3}$ Ⓒ 5
Ⓑ $2\sqrt{6}$ Ⓓ $\pi + 1$

8. What number is shown on the graph?



Ⓐ $\pi + 3$ Ⓒ $\sqrt{20} + 2$
Ⓑ $\sqrt{4} + 2.5$ Ⓓ $6.\overline{14}$

9. Which is in order from least to greatest?

Ⓐ $3.3, \frac{10}{3}, \pi, \frac{11}{4}$ Ⓒ $\pi, \frac{10}{3}, \frac{11}{4}, 3.3$
Ⓑ $\frac{10}{3}, 3.3, \frac{11}{4}, \pi$ Ⓓ $\frac{11}{4}, \pi, 3.3, \frac{10}{3}$

Mini-Task

10. The volume of a cube is given by $V = x^3$, where x is the length of an edge of the cube. The area of a square is given by $A = x^2$, where x is the length of a side of the square. A given cube has a volume of 1728 cubic inches.

a. Find the length of an edge.

____

b. Find the area of one side of the cube.

____

c. Find the surface area of the cube.

____

d. What is the surface area in square feet?

____

Real Numbers

MODULE 1

ESSENTIAL QUESTION

How can you use real numbers to solve real-world problems?

Real-World Video

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Living creatures can be classified into groups. The sea otter belongs to the kingdom Animalia and class Mammalia. Numbers can also be classified into groups such as rational numbers and integers.

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Complete these exercises to review skills you will need for this module.

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Find the Square of a Number

EXAMPLE Find the square of $\frac{2}{3}$.

$\frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3}$ *Multiply the number by itself.*

$= \frac{4}{9}$ *Simplify.*

Find the square of each number.

1. 7 ______ **2.** 21 ______ **3.** -3 ______ **4.** $\frac{4}{5}$ ______

5. 2.7 ______ **6.** $-\frac{1}{4}$ ______ **7.** -5.7 ______ **8.** $1\frac{2}{5}$ ______

Exponents

EXAMPLE $5^3 = 5 \times 5 \times 5$ *Use the base, 5, as a factor 3 times.*

$= 25 \times 5$ *Multiply from left to right.*

$= 125$

Simplify each exponential expression.

9. 9^2 ______ **10.** 2^4 ______ **11.** $\left(\frac{1}{3}\right)^2$ ______ **12.** $(-7)^2$ ______

13. 4^3 ______ **14.** $(-1)^5$ ______ **15.** 4.5^2 ______ **16.** 10^5 ______

Write a Mixed Number as an Improper Fraction

EXAMPLE $2\frac{2}{5} = 2 + \frac{2}{5}$ *Write the mixed number as a sum of a whole number and a fraction.*

$= \frac{10}{5} + \frac{2}{5}$ *Write the whole number as an equivalent fraction with the same denominator as the fraction in the mixed number.*

$= \frac{12}{5}$ *Add the numerators.*

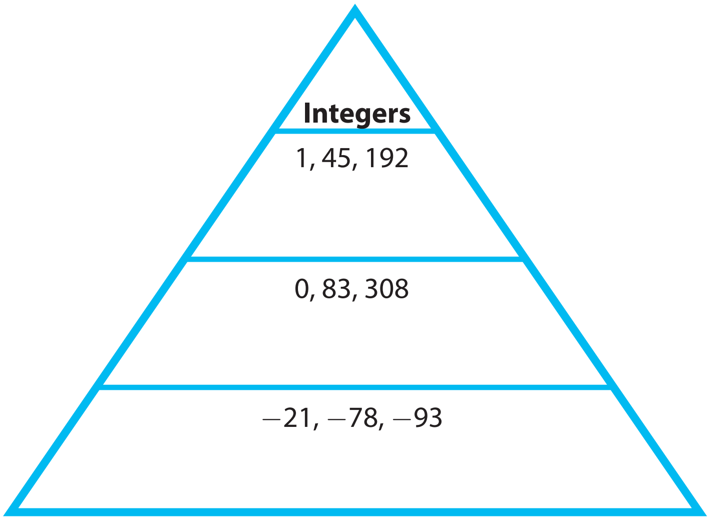
Write each mixed number as an improper fraction.

17. $3\frac{1}{3}$ ______ **18.** $1\frac{5}{8}$ ______ **19.** $2\frac{3}{7}$ ______ **20.** $5\frac{5}{6}$ ______

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You can put more than one word in each section of the triangle.



Understand Vocabulary

Complete the sentences using the preview words.

1. One of the two equal factors of a number is a _______________.
2. A ____________________ has integers as its square roots.
3. The ___________________________ is the nonnegative square root of a number.

Vocabulary

Review Words

- integers *(enteros)*
- ✔ negative numbers *(números negativos)*
- ✔ positive numbers *(números positivos)*
- ✔ whole number *(número entero)*

Preview Words

- cube root *(raiz cúbica)*
- irrational numbers *(número irracional)*
- perfect cube *(cubo perfecto)*
- perfect square *(cuadrado perfecto)*
- principal square root *(raíz cuadrada principal)*
- rational number *(número racional)*
- real numbers *(número real)*
- repeating decimal *(decimal periódico)*
- square root *(raíz cuadrada)*
- terminating decimal *(decimal finito)*

Active Reading

Layered Book Before beginning the lessons in this module, create a layered book to help you learn the concepts in this module. Label the flaps "Rational Numbers," "Irrational Numbers," "Square Roots," and "Real Numbers." As you study each lesson, write important ideas such as vocabulary, models, and sample problems under the appropriate flap.

MODULE 1

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.NS.1.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Key Vocabulary

rational number *(número racional)*
A number that can be expressed as a ratio of two integers.

irrational number *(número irracional)*
A number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal.

What It Means to You

You will recognize a number as rational or irrational by looking at its fraction or decimal form.

UNPACKING EXAMPLE 8.NS.1.1

Classify each number as rational or irrational.

$0.\overline{3} = \frac{1}{3}$ $\qquad$ $0.25 = \frac{1}{4}$

These numbers are rational because they can be written as ratios of integers or as repeating or terminating decimals.

$\pi \approx 3.141592654\ldots$ $\qquad$ $\sqrt{5} \approx 2.236067977\ldots$

These numbers are irrational because they cannot be written as ratios of integers or as repeating or terminating decimals.

FL 8.NS.1.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

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What It Means to You

You will learn to estimate the values of irrational numbers.

UNPACKING EXAMPLE 8.NS.1.2

Estimate the value of $\sqrt{8}$.

8 is not a perfect square. Find the two perfect squares closest to 8.

8 is between the perfect squares 4 and 9.
So $\sqrt{8}$ is between $\sqrt{4}$ and $\sqrt{9}$.
$\sqrt{8}$ is between 2 and 3.

8 is close to 9, so $\sqrt{8}$ is close to 3.
$2.8^2 = 7.84$ $\qquad$ $2.85^2 = 8.1225$ $\qquad$ $2.9^2 = 8.41$
$\sqrt{8}$ is between 2.8 and 2.9, but closer to 2.8.
A good estimate for $\sqrt{8}$ is 2.8.

LESSON 1.1

Rational and Irrational Numbers

FL 8.NS.1.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; ... *Also 8.NS.1.2, 8.EE.1.2*

ESSENTIAL QUESTION

How do you rewrite rational numbers and decimals, take square roots and cube roots, and approximate irrational numbers?

Expressing Rational Numbers as Decimals

A **rational number** is any number that can be written as a ratio in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not 0. Examples of rational numbers are 6 and 0.5.

6 can be written as $\frac{6}{1}$.   0.5 can be written as $\frac{1}{2}$.

Every rational number can be written as a terminating decimal or a repeating decimal. A **terminating decimal**, such as 0.5, has a finite number of digits. A **repeating decimal** has a block of one or more digits that repeat indefinitely.

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EXAMPLE 1   FL 8.NS.1.1

Write each fraction as a decimal.

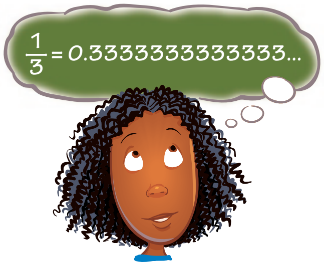
A $\frac{1}{4}$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Remember that the fraction bar means "divided by." Divide the numerator by the denominator.

Divide until the remainder is zero, adding zeros after the decimal point in the dividend as needed.

$\frac{1}{4} = 0.25$



B $\frac{1}{3}$

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

Divide until the remainder is zero or until the digits in the quotient begin to repeat.

Add zeros after the decimal point in the dividend as needed.

When a decimal has one or more digits that repeat indefinitely, write the decimal with a bar over the repeating digit(s).

$\frac{1}{3} = 0.\overline{3}$

My Notes

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YOUR TURN

Write each fraction as a decimal.

1. $\frac{5}{11}$ ________ **2.** $\frac{1}{8}$ ________ **3.** $2\frac{1}{3}$ ________

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Expressing Decimals as Rational Numbers

You can express terminating and repeating decimals as rational numbers.

EXAMPLE 2

FL 8.NS.1.1

My Notes

Write each decimal as a fraction in simplest form.

A 0.825

The decimal 0.825 means "825 thousandths." Write this as a fraction.

$\frac{825}{1000}$ *To write "825 thousandths", put 825 over 1000.*

Then simplify the fraction.

$\frac{825 \div 25}{1000 \div 25} = \frac{33}{40}$ *Divide both the numerator and the denominator by 25.*

$0.825 = \frac{33}{40}$

B $0.\overline{37}$

Let $x = 0.\overline{37}$. The number $0.\overline{37}$ has 2 repeating digits, so multiply each side of the equation $x = 0.\overline{37}$ by 10^2, or 100.

$x = 0.\overline{37}$

$(100)x = 100(0.\overline{37})$

$100x = 37.\overline{37}$ *100 times $0.\overline{37}$ is $37.\overline{37}$.*

Because $x = 0.\overline{37}$, you can subtract x from one side and $0.\overline{37}$ from the other.

$$\begin{array}{rr} 100x = & 37.\overline{37} \\ -x & -0.\overline{37} \\ \hline 99x = & 37 \end{array}$$

$37.\overline{37}$ minus $0.\overline{37}$ is 37.

Now solve the equation for x. Simplify if necessary.

$\frac{99x}{99} = \frac{37}{99}$ *Divide both sides of the equation by 99.*

$x = \frac{37}{99}$

YOUR TURN

Write each decimal as a fraction in simplest form.

4. 0.12 ________ **5.** $0.\overline{57}$ ________ **6.** 1.4 ________

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Finding Square Roots and Cube Roots

The **square root** of a positive number p is x if $x^2 = p$. There are two square roots for every positive number. For example, the square roots of 36 are 6 and -6 because $6^2 = 36$ and $(-6)^2 = 36$. The square roots of $\frac{1}{25}$ are $\frac{1}{5}$ and $-\frac{1}{5}$. You can write the square roots of $\frac{1}{25}$ as $\pm\frac{1}{5}$. The symbol $\sqrt{}$ indicates the positive, or **principal square root**.

A number that is a **perfect square** has square roots that are integers. The number 81 is a perfect square because its square roots are 9 and -9.

The **cube root** of a positive number p is x if $x^3 = p$. There is one cube root for every positive number. For example, the cube root of 8 is 2 because $2^3 = 8$. The cube root of $\frac{1}{27}$ is $\frac{1}{3}$ because $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$. The symbol $\sqrt[3]{}$ indicates the cube root.

A number that is a **perfect cube** has a cube root that is an integer. The number 125 is a perfect cube because its cube root is 5.

EXAMPLE 3

FL 8.EE.1.2

Solve each equation for x.

A $x^2 = 121$

$x^2 = 121$ — *Solve for x by taking the square root of both sides.*

$x = \pm\sqrt{121}$ — *Apply the definition of square root.*

$x = \pm 11$ — *Think: What numbers squared equal 121?*

The solutions are 11 and -11.

B $x^2 = \frac{16}{169}$

$x^2 = \frac{16}{169}$ — *Solve for x by taking the square root of both sides.*

$x = \pm\sqrt{\frac{16}{169}}$ — *Apply the definition of square root.*

$x = \pm\frac{4}{13}$ — *Think: What numbers squared equal $\frac{16}{169}$?*

The solutions are $\frac{4}{13}$ and $-\frac{4}{13}$.

Math Talk
Mathematical Practices

Can you square an integer and get a negative number? What does this indicate about whether negative numbers have square roots?